

**AUTOMATION - CONTROL  
AND INDUSTRIAL ENGINEERING SERIES**



# **Optimization in Engineering Sciences**

*Exact Methods*

**Pierre Borne, Dumitru Popescu  
Florin Gh. Filip and Dan Stefanoiu**

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## Table of Contents

<b>Foreword</b> .....	<b>ix</b>
<b>Preface</b> .....	<b>xi</b>
<b>List of Acronyms</b> .....	<b>xiii</b>
<b>Chapter 1. Linear Programming</b> .....	<b>1</b>
1.1. Objective of linear programming.....	1
1.2. Stating the problem.....	1
1.3. Lagrange method.....	4
1.4. Simplex algorithm.....	5
1.4.1. Principle.....	5
1.4.2. Simplicial form formulation.....	5
1.4.3. Transition from one simplicial form to another.....	7
1.4.4. Summary of the simplex algorithm.....	9
1.5. Implementation example.....	11
1.6. Linear programming applied to the optimization of resource allocation.....	13
1.6.1. Areas of application.....	13
1.6.2. Resource allocation for advertising.....	13
1.6.3. Optimization of a cut of paper rolls.....	16
1.6.4. Structure of linear program of an optimal control problem.....	17
<b>Chapter 2. Nonlinear Programming</b> .....	<b>23</b>
2.1. Problem formulation.....	23
2.2. Karush–Kuhn–Tucker conditions.....	24
2.3. General search algorithm.....	26
2.3.1. Main steps.....	26
2.3.2. Computing the search direction.....	29

2.3.3. Computation of advancement step.....	33
2.4. Monovariate methods.....	33
2.4.1. Coggin’s method (of polynomial interpolation) .....	34
2.4.2. Golden section method .....	36
2.5. Multivariable methods .....	39
2.5.1. Direct search methods .....	39
2.5.2. Gradient methods.....	57
<b>Chapter 3. Dynamic Programming.....</b>	<b>101</b>
3.1. Principle of dynamic programming .....	101
3.1.1. Stating the problem.....	101
3.1.2. Decision problem.....	101
3.2. Recurrence equation of optimality .....	102
3.3. Particular cases .....	104
3.3.1. Infinite horizon stationary problems.....	104
3.3.2. Variable horizon problem .....	104
3.3.3. Random horizon problem .....	104
3.3.4. Taking into account sum-like constraints .....	105
3.3.5. Random evolution law .....	106
3.3.6. Initialization when the final state is imposed.....	106
3.3.7. The case when the necessary information is not always available ....	107
3.4. Examples.....	107
3.4.1. Route optimization .....	107
3.4.2. The smuggler problem.....	109
<b>Chapter 4. Hopfield Networks.....</b>	<b>115</b>
4.1. Structure.....	115
4.2. Continuous dynamic Hopfield networks.....	117
4.2.1. General problem .....	117
4.2.2. Application to the traveling salesman problem .....	121
4.3. Optimization by Hopfield networks, based on simulated annealing.....	123
4.3.1. Deterministic method .....	123
4.3.2. Stochastic method.....	125
<b>Chapter 5. Optimization in System Identification.....</b>	<b>131</b>
5.1. The optimal identification principle.....	131
5.2. Formulation of optimal identification problems .....	132
5.2.1. General problem .....	132
5.2.2. Formulation based on optimization theory .....	133
5.2.3. Formulation based on estimation theory (statistics) .....	136
5.3. Usual identification models .....	138

5.3.1. General model .....	138
5.3.2. Rational input/output (RIO) models .....	140
5.3.3. Class of autoregressive models (ARMAX) .....	142
5.3.4. Class of state space representation models .....	145
5.4. Basic least squares method .....	146
5.4.1. LSM type solution .....	146
5.4.2. Geometric interpretation of the LSM solution.....	151
5.4.3. Consistency of the LSM type solution.....	154
5.4.4. Example of application of the LSM for an ARX model .....	157
5.5. Modified least squares methods .....	158
5.5.1. Recovering lost consistency .....	158
5.5.2. Extended LSM.....	162
5.5.3. Instrumental variables method.....	164
5.6. Minimum prediction error method.....	168
5.6.1. Basic principle and algorithm.....	168
5.6.2. Implementation of the MPEM for ARMAX models .....	171
5.6.3. Convergence and consistency of MPEM type estimations.....	174
5.7. Adaptive optimal identification methods .....	175
5.7.1. Accuracy/adaptability paradigm.....	175
5.7.2. Basic adaptive version of the LSM.....	177
5.7.3. Basic adaptive version of the IVM.....	182
5.7.4. Adaptive window versions of the LSM and IVM.....	183
<b>Chapter 6. Optimization of Dynamic Systems .....</b>	<b>191</b>
6.1. Variational methods.....	191
6.1.1. Variation of a functional.....	191
6.1.2. Constraint-free minimization.....	192
6.1.3. Hamilton canonical equations.....	194
6.1.4. Second-order conditions .....	195
6.1.5. Minimization with constraints .....	195
6.2. Application to the optimal command of a continuous process, maximum principle .....	196
6.2.1. Formulation .....	196
6.2.2. Examples of implementation .....	198
6.3. Maximum principle, discrete case.....	206
6.4. Principle of optimal command based on quadratic criteria .....	207
6.5. Design of the LQ command .....	210
6.5.1. Finite horizon LQ command.....	210
6.5.2. The infinite horizon QL command .....	217
6.5.3. Robustness of the LQ command.....	221
6.6. Optimal filtering.....	224
6.6.1. Kalman–Bucy predictor.....	225

6.6.2. Kalman–Bucy filter .....	231
6.6.3. Stability of Kalman–Bucy estimators .....	234
6.6.4. Robustness of Kalman–Bucy estimators .....	235
6.7. Design of the LQG command .....	239
6.8. Optimization problems connected to quadratic linear criteria .....	245
6.8.1. Optimal control by state feedback .....	245
6.8.2. Quadratic stabilization .....	248
6.8.3. Optimal command based on output feedback .....	249
<b>Chapter 7. Optimization of Large-Scale Systems</b>	
251	
7.1. Characteristics of complex optimization problems .....	251
7.2. Decomposition techniques .....	252
7.2.1. Problems with block-diagonal structure .....	253
7.2.2. Problems with separable criteria and constraints .....	267
7.3. Penalization techniques .....	283
7.3.1. External penalization technique .....	284
7.3.2. Internal penalization technique .....	285
7.3.3. Extended penalization technique .....	286
<b>Chapter 8 Optimization and Information Systems.....</b>	<b>289</b>
8.1. Introduction .....	289
8.2. Factors influencing the construction of IT systems .....	290
8.3. Approaches .....	292
8.4. Selection of computing tools .....	296
8.5. Difficulties in implementation and use .....	297
8.6. Evaluation .....	297
8.7. Conclusions .....	298
<b>Bibliography.....</b>	<b>299</b>

## Foreword

The optimization theory field is already well defined, strong and mature, with plenty of theoretical results and remarkable applications. Nowadays, it takes courage to publish a new book on classical optimization issues. Although it is said that anyone can conceive a new optimization technique, outperforming the existing algorithms in terms of convergence speed and efficient implementation is rather difficult. However, improvements should be possible.

What makes this book interesting, and original at the same time, is something that is often missing from publications of quality scientific literature: the engineering point of view. As Albert Einstein said so well, it is quite sad to see how a beautiful theory is destroyed by an ugly reality. In this spirit, optimization theory has plenty of pure theoretical results that are quite impossible to transform into efficient numerical procedures to be employed later in real applications. However, the milestone of this book is, seemingly, the optimization algorithm for the benefit of application.

The authors succeeded in describing quite a large panoply of optimization techniques, from simple ones like linear or dynamic programming, to complex ones including nonlinear programming, large-scale systems, system identification or automatic control strategies. Of course, no-one can encompass in a single volume all the optimization methods that the authors refer here to as “exact”, i.e. non-heuristic, or stochastic. For example, the recent group of Linear Matrix Inequality (LMI) optimization techniques, based on the interior point methods, is not presented here. However, I assume that the final goal to fulfill here was not to cover all possible topics in optimization, as in a treatise. The authors rather intended to meet the engineering need for clear and efficient optimization procedures ready to be implemented and, moreover, easy to adapt to specific applications. In spite of optimization toolboxes or dynamic link libraries that can be found on various software platforms, the user is faced with two major problems when approaching



applications that require optimization of some criteria. First of all, he/she does not know very well the meaning of input arguments to be set for a function implementing some optimization technique. This book enlightens the user in this aim, by revealing how to configure the numerical procedure associated with each optimization technique. Second, he/she could not modify the optimization function if some application requirements are to be met. On the contrary, very often, problems within specific applications are reformulated, in order to adapt to some available optimization procedure, which, of course, could change the initial nature of those applications. This book describes the steps of each algorithm in a clear and concise manner, so that anyone can implement it in some particular way, if necessary.

The methods described in the book include: linear programming with various implementations, nonlinear programming, dynamic programming with various application examples, Hopfield networks, optimization in systems identification, optimization of dynamic system with particular application to process control, optimization of large-scale and complex systems using decomposition techniques, optimization and information systems.

As described above, the reader may understand that the book is just an optimization algorithms compendium, which is not true at all. It is much more than that. For each algorithm, where possible, a sound analysis concerning its foundation, convergence, complexity and efficiency is presented. Easy to follow examples also exist, where possible. Most of the numerical procedures introduced here are improved comparing to the original or other improved procedures found in the scientific literature.

As a final word, I am pleased to see that exact optimization methods could be improved and, moreover, help the engineer, regardless of the fields of activity, to better understand them, how to apply them, their limitations, etc. Definitely, the authors were inspired to write such a book, which, I hope, will be welcomed both by the scientific community and practitioners.

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November 2012

## Preface

The purpose of this book is to introduce the most important methods of static and dynamic optimization, from an engineering point of view.

The methods are *exact*, in the sense that optimum solutions are searched by means of accurate, deterministic numerical algorithms, the convergence being soundly proven for most of them.

In order to focus on the optimization algorithms and to make the presentation friendly, the proofs of various results are often not developed. However, some remarks or short rationales regarding the principles of various proposed algorithms, sometimes with additional references allowing the interested reader to explore the optimization topics in depth, are given.

When the optimization algorithms are not too complex, some easy to follow and reproducible implementation examples are presented.

The methods described within the book include:

- linear programming with various implementations;
- nonlinear programming, which is a particularly important topic, given the wide variety of existing algorithms;
- dynamic programming with various application examples;
- Hopfield networks;
- optimization in systems identification;
- optimization of dynamic systems with particular application to process control;
- optimization of large-scale and complex systems;
- optimization and information systems.

Optimization techniques for difficult problems implementing metaheuristic, stochastic and suboptimal approaches will be addressed in a different book.

This book was produced within the framework of the European FP7 project ERRIC (*Empowering Romanian Research on Intelligent Information Technology*), contract FP7-REGPOT-2010-1/264207 and developed in cooperation between French and Romanian scientists.

*Pierre Borne, Dumitru Popescu, Floria G. Filip and Dan Stefanoiu*  
Lille and Bucharest  
November 2012

The purpose of this book is to present the main methods of static and dynamic optimization. It has been written within the framework of the European Union project – ERRIC (Empowering Romanian Research on Intelligent Information Technologies), funded by the EU's FP7 Research Potential program and developed in cooperation between French and Romanian teaching researchers.

Through the principles of various proposed algorithms (with additional references) this book allows the interested reader to explore various methods of implementation such as linear programming, nonlinear programming – particularly important given the wide variety of existing algorithms, dynamic programming with various application examples and Hopfield networks. The book examines optimization in relation to systems identification; optimization of dynamic systems with particular application to process control; optimization of large scale and complex systems; optimization and information systems.

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